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## COMMENTS ON PRESENTATION BY PAUL COX

Walter T. Federer  
Mathematics Research Center, United States Army  
University of Wisconsin, Madison, Wisconsin

The paper presented by Mr. Cox is written in a somewhat provocative manner. I appreciate this style of presentation as it affords the Panel ample opportunity to illustrate several statistical points.

The first point I wish to make relates to the definition and use of terms in current statistical literature. There is a tendency in statistical literature for vague and imprecise usage of such terms as the design of experiments, analysis of variance, error rate, etc. It is instructive and useful to define and to use words or phrases in a specified manner. Any departure from specificity should be described. Personally, I would prefer to use definitions of the following form:

i) Experimental design (or experiment design) - The arrangement of the observations in the experimental area or space or the procedure for obtaining the observations in an experiment.

ii) Treatment design - The arrangement or selection of treatments for the experiment (e. g., the selection of levels and combinations of factors in factorial experiments, etc.)

iii) Determination of sample size - The number of observations necessary to achieve a prescribed objective. (Authors of some ranking procedures papers refer to the determination of numbers of observations as the design of experiment rather than as the determination of sample size.)

iv) Analysis of variance - The partitioning of the sum of squares into component parts. (One segment of statistical literature utilizes the term analysis of variance to be synonymous with an  $F$  test while another segment utilizes this term to refer to the estimation of variance components and so it goes.)

v) Analysis of experimental data - This term includes the last above but not vice versa. It refers to all statistical computations relevant to a set of experimental data. An analysis of experimental data refers to the reduction of data to summary form and is useful in, but does not replace, the interpretation of experimental results. The interpretation of statistical results must be made in light of the objectives, conditions, and related circumstances of the experimental results.

vi) Significance level - Type I error = size of the test =  $\alpha$ , have all been used to refer to the same thing but unfortunately nothing is said about the base for computing " $\alpha$ ".

vii) Valid estimate of the error variance - Fisher has defined this term but unfortunately many statistical writers by-pass this important concept with the phrase "given that  $\sigma^2$  is the error variance." In much of experimentation the definition of error variance cannot be so glibly by-passed, but requires a thorough knowledge of the experimental conditions.

We could go on with other terms but now let us return to Mr. Cox's paper. The title of the paper is "Statistical Design of Experiment for Continuous Data"; it deals only with the analysis of experimental results with no reference either to the experimental or treatment design as defined above. Mr. Hartley has discussed some considerations to be given to the treatment design for experiments with specified objectives. Mr. Lucas will, I hope, make some comments about the actual experimental design used in this study and illustrate where confounding has taken place. Mr. Cox's paper is concerned with what to do with a set of data and not with how to obtain the data. He has raised a number of questions but rather than address myself to the specific question I prefer to proceed in another manner which, I hope, will furnish answers to or illustrate the relevance of the questions.

As Messrs. Grubbs, Greenberg, Hartley, and Schneiderman have already stressed we must first set up a Mathematical Model for the data which will be consistent with the experimental and treatment designs and with the nature and objectives of the experiment. For example, let us suppose that thrust =  $y$ , may be characterized by the following:

$$y = f(\underline{\epsilon}, \underline{t}, \underline{\theta})$$

where the response variable  $y$  is a function of error components denoted by the vector  $\underline{\epsilon}$ , of time components denoted by the vector  $\underline{t}$ , and of a set of parameters denoted by the vector  $\underline{\theta}$ . Our first job then is to define to nature of the function. If we are totally ignorant of the response curve then we could use a form of polynomial regression as follows:

$$E(y) = \sum_{i=0}^b \beta_i t^i$$

where  $\beta_i$  is the  $i^{\text{th}}$  regression coefficient and  $t$  the time variable. After we are satisfied that a suitable mathematical formulation of the problem has been made, the parameters of the response curve are estimated. The analysis of the estimates may be made using the results of R. A. Fisher (Jour. Agric. Sci. 11:107, 1921 and Phil. Trans. Roy. Soc. B, 213:89, 1925) and others. Also, multivariate analysis procedures may be pursued for summarizing the results for many estimates of a set of parameters. For example, if it is desired to discriminate between response curves, then an a priori or an a posteriori (These terms are not reserved solely for use by Bayesians.) weighting of coefficients in the discriminate function may be utilized.

As a part of the characterization of the model and of the problem it should be determined if the total response curve segments of the total curve, or specified points (e.g. points of inflection) on the curve are of interest. After this has been specified then the statistician proceeds with the estimation problems. Haziness on form or type of response desired leads to a confusion of issues.

One specific question raised by Mr. Cox related to the sample size  $N$  for response curves for continuous data. Now if the data are truly continuous  $N = \text{infinity}$ , but we all know that the recording machine records an impulse over a measurable period of time, say one-tenth of a second. In any event  $N$  is very large. Several of the previous Panel speakers have discussed the non-independence of two successive impulses or recordings by a recording machine. However, I wonder about the relevance of this since we use, or should use, these values only to estimate the parameters in the response curve. This procedure is, or should be, repeated for many response curves and the variation among response curves treated alike forms a basis for the variances and covariances among the estimates of parameters where each response curve represents but one observation.

At this point I do not see the importance of obtaining a variance of a single response curve. However, if such is desired, then as an approximation I would suggest segmentation of the total curves into small segments of time where small is such that the estimates are relatively unaffected by smaller segmentation. Course groupings could affect the results considerably. Some account may need to be taken of the relationship among adjoining segments as described by Messrs. Greenberg and Hartley.

The response curves presented in the paper bother me somewhat. Frankly, I believe (i) that the curves in Figure 3 are not very fictitious, (ii) that the area under each curve is relatively constant from the conservation of mass theory, (iii) that a heart-to-heart talk with the physicists and engineers would do much to simplify the nature of the problem, and (iv) that maybe Mr. Cox should be considering acceleration =  $z$  instead of thrust =  $y$ .

Summed up this means that I would want some education in this area before any analyses would be performed on thrust or any other data. It may be possible to reparameterize the problem by using a function of the time variable instead of the time variable itself. Some simple function such as  $\log t$  might suffice.